FAILING YELLOW

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Lemma 0.1. *Consider a* SPRT(elo0, elo1) *under the BayesElo model. Let* d *be the draw ratio and let* dr *be the* LLR *associated with a draw. Assume* $\mathrm{dr} < 0$*. Let* A *be the* LLR *associated with accepting* H0*. Put*

$$N = \frac{2A}{(1+d)\,\mathrm{dr}}\,. \tag{1}$$

If the test fails then it is approximately[1] *true that it fails yellow if and only if the number of games is* $\geq N$*.*

Example 0.2. Assume SPRT$(0, 5)$, $\alpha = \beta = 0.05$, drawelo$= 270$. In this case, to fail yellow, we need ≥ 29892 games (approximately).

Remark 0.3. It follows from the lemma that a SPRT with an additional pass condition associated with "failing yellow" is almost the same as a "truncated SPRT". In itself a truncated SPRT is sound but it has altered Type I/II error probabilities. In fact truncated SPRTs are mathematically quite difficult to handle and one should only use them if tests of bounded length are absolutely essential.

Proof of Lemma 0.1. We first introduce some notations. Let W, D, L be the win, draw, loss count with $N = W + D + L$ being the total number of games. Let wr, dr, lr be the LLR for win, draw, loss respectively. We do not yet assume the BayesElo model.

We have

$$N = W + D + L = 2W + D - E = 2W + Nd - E$$

with $E = W - L$ and approximating D by Nd. Hence

$$\begin{aligned} W &= \frac{1}{2}(N(1-d) + E) \\ D &= dN \\ L &= \frac{1}{2}(N(1-d) - E) \end{aligned}$$

Assume a test has just failed. Failing means (ignoring overshoots)

$$W\,\mathrm{wr} + D\,\mathrm{dr} + L\,\mathrm{lr} = A$$

[1]The proof uses some approximations and I have not bothered to bound the corresponding errors.

Substituting the expressions for W, D, L this yields

$$\begin{aligned} A &= \frac{1}{2}(N(1-d)+E)\,\mathrm{wr} + Nd\,\mathrm{dr} + \frac{1}{2}(N(1-d)-E)\,\mathrm{lr} \\ &= N\left(\frac{1}{2}(1-d)\,\mathrm{wr} + d\,\mathrm{dr} + \frac{1}{2}(1-d)\,\mathrm{lr}\right) + E\left(\frac{1}{2}\,\mathrm{wr} - \frac{1}{2}\,\mathrm{lr}\right) \\ &= N\left(\frac{1}{2}(1-d)(\mathrm{wr}+\mathrm{lr}) + d\,\mathrm{dr}\right) + E\left(\frac{1}{2}(\mathrm{wr}-\mathrm{lr})\right) \end{aligned}$$

We now derive the condition for the test to fail yellow. Failing yellow means $E \geq 0$. Since $\mathrm{wr} - \mathrm{lr} \geq 0$ we obtain

$$N\left(\frac{1}{2}(1-d)(\mathrm{wr}+\mathrm{lr}) + d\,\mathrm{dr}\right) \leq A$$

If we are using the Bayes-Elo model then $\mathrm{wr} + \mathrm{lr} = \mathrm{dr}$ so that we get the simple formula

$$N \geq \frac{2A}{(1+d)\,\mathrm{dr}} \tag{2}$$

taking into account our hypothesis $\mathrm{dr} < 0$. □